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Metamaterials and Transformation Optics

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14. ABSTRACT

Several US collaborations were reinforce during this period with several jointly authored papers appearing, mainly concerning the applications of metamaterials to cloaking. It is evident that there has been considerable exchange of ideas between the US groups and the Imperial College London group. This continues, and further exchange of personnel is planned for 2011.

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Metamaterials and transformation optics

report on activities February 2010 – January 2011

JB Pendry

During the second year of activities there were 4 separate visits to the USA:

- Visit to Princeton University for discussions and lectures on metamaterials: Monday 5 to Saturday 10 April 2010.
- Discussion with David Smith at Duke University; Triservice Metamaterial Basic Research Program Review, Virginia Beach: Friday 14 to Friday 28 May 2010.
- Visit to Johns Hopkins University, Baltimore for discussion and lecture on metamaterials: Wednesday 22 to Friday 24 September 2011.
- Visit to 'Intellectual Ventures' Seattle to consult and to UCSD to lecture on metamaterials: Monday 1 to Saturday 6 November 2010.

This makes up the one month agreed time in the USA.

My several US collaborations were reinforce during this period with several jointly authored papers appearing, mainly concerning the applications of metamaterials to cloaking.

In a paper joint with the Duke team we examined

Transformation optics (TO) is a recently appreciated tool for the design of complex media with unique wave propagation properties . Introduced in the present context as a computational technique to extend the utility of finite difference and finite-element codes, TO has become widely appreciated for its generality and ability to design structures that manipulate waves with unprecedented control. The main tool associated with TO is that of coordinate transformations, in which isotropic space is conceptually warped or otherwise distorted as a means of guiding the trajectories of waves. The coordinate transformation that results in some desired functionality can then be used to determine the properties of a physical medium in which waves will behave as if they were propagating in the warped space. The TO method can be applied to any linear waves for which the underlying equations exhibit form invariance under coordinate transformations. Maxwell's equations, for example, are generally form-invariant, so that coordinate transformations can equivalently be implemented as spatially varying, anisotropic constitutive parameters (i.e. the electric permittivity and the magnetic permeability).

TO is an exact tool developed by the Duke team and myself for the design of many devices and in particular has been applied to design the first working cloak. However less exact competitive schemes have been proposed (but not implemented) and in [6] we examined several cloaking schemes.

In the figure below taken from our New J. Phys. paper we illustrate the practical problems of a cylindrical cloak whose function is to shrink the hidden object to a very thin wire. Unfortunately even thin wires can scatter quite strongly and are only invisible when infinitely thin – something that requires infinite precision of the cloak. To some degree this problem can be avoided by choosing the correct inner lining for the cloak (top). The graphs show the total scattering cross section normalised to the physical cross section as a function of wavelength.

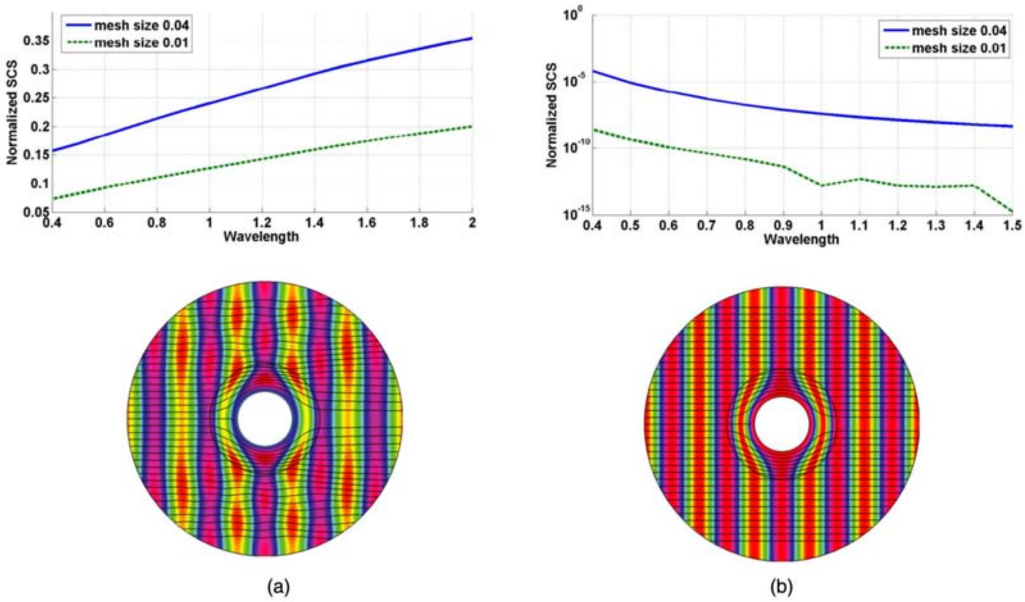


Figure 1. Two-dimensional perfect cloak defined by equation (1) with the linear transformation (3). The graphs show the total scattering cross section assuming (a) hard and (b) soft boundary conditions on the inner surface of the cloak, as a function of the wavelength. The images underneath the graphs show the electric field distributions at the wavelength $\lambda = 0.4$. The cloak inner and outer radii are $a = 0.2$ and $b = 0.4$, respectively. The normalized SCS is defined as the ratio $\sigma_{sc}/\sigma_{geom}$, where $\sigma_{geom} = 2a$ is the geometric width of the cloaked region. Numerical simulation performed with COMSOL Multiphysics.

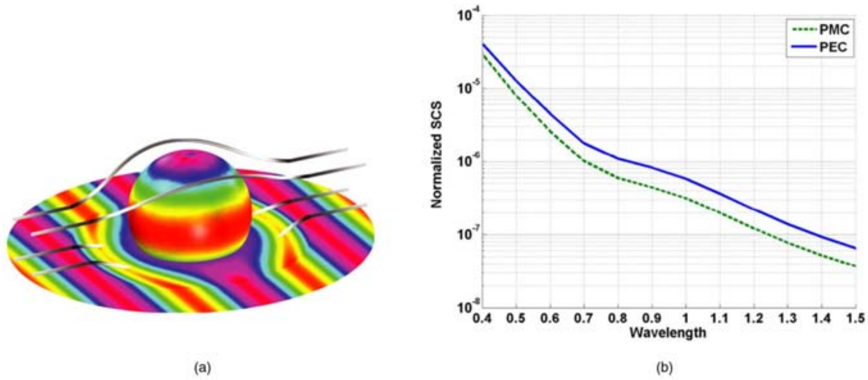


Figure 2. Three-dimensional perfect cloak defined by equation (2) with the linear transformation (3). Plots show (a) field profile and streamlines of electromagnetic flux at the wavelength $\lambda = 0.4$ and (b) the total SCS with hard (PEC) and soft (PMC) boundary conditions, as a function of the wavelength. The cloak inner and outer radii are $a = 0.2$ and $b = 0.4$.

The figure in the lower half of the pane shows scattering from a 3D cloak which is intrinsically more efficient that a cylindrical cloak as evidenced by the lack of sensitivity to the inner boundary conditions.

A competing design process of cloaks proposed by Leonhardt uses an approximation to Maxwell's equations in the form of the Helmholtz equation. This neglects the near field components of the waves and produces by means of a conformal transformation cloak that composed of an isotropic refractive index, but that is infinite in extent. We Of course any practical cloak has to be finite so we explored the effect on truncating the cloak at various radii. Our computations are displayed below. Unfortunately the truncation has serious consequences for the effectiveness of the cloak as the scattering cross section relative to the uncloaked object, increases as we attempt to converge to the infinite cloak by taking successively larger approximations. As a result we conclude that this is not a practical design.

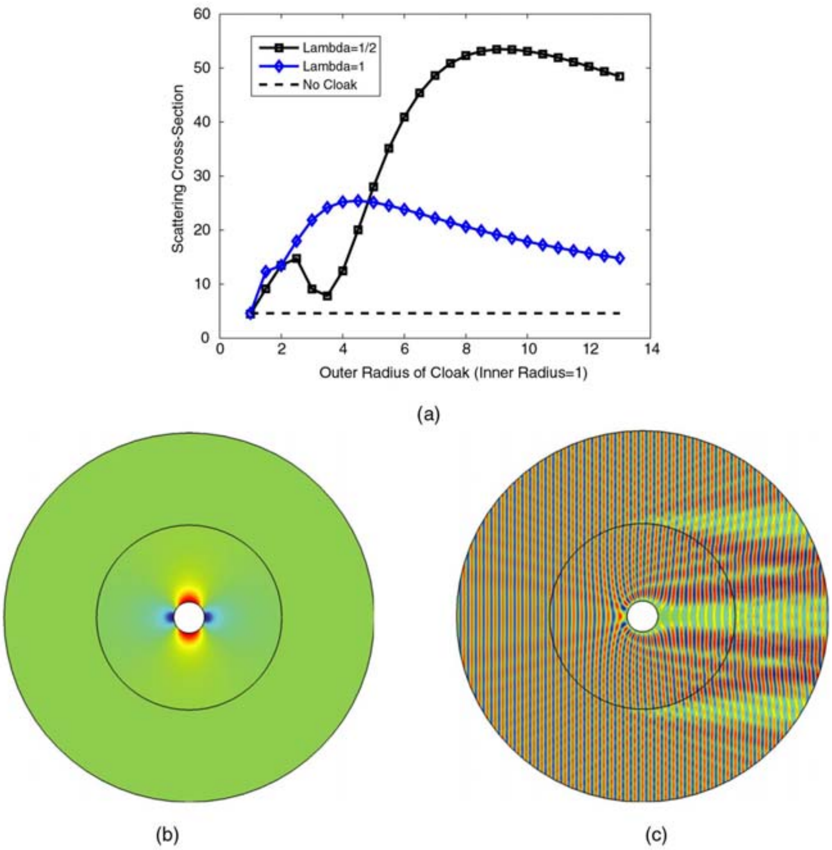


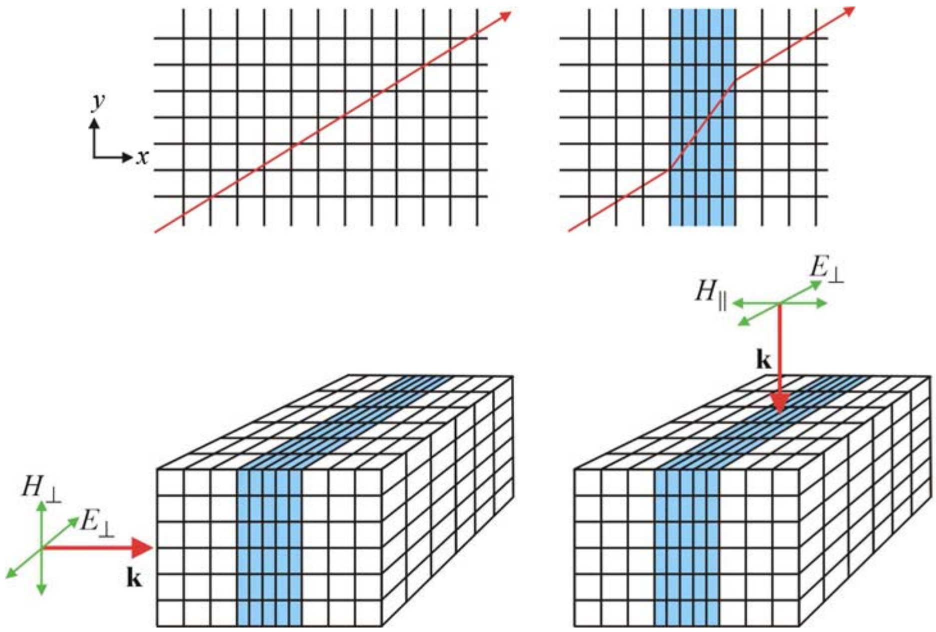
Figure 9. Full-wave simulations of the conformal cloak with various sizes. (a) The SCS (in arbitrary units) as a function of the cloak outer radius b for two wavelengths, $\lambda = 1$ and 0.5, assuming $a = 1$. (b) The index profile for a cloak with the shape aspect ratio $b/a = 6$. The refractive index mismatch at the boundary $r = b$ is less than 10%, yet it affects the cross section. (c) Full-wave simulation for the same cloak with $b/a = 6$. Refraction at the outer boundary and shadowing around the core cylinder are evident; these effects contribute to the SCS.

In a second paper, joint with the Duke team [9], we reviewed the transformation optics technology in the manner of a tutorial with the aim of spreading the technology more widely. We were invited to publish in an influential proceedings. The paper begins with a simple explanation of the technique:

To give a flavour of how the scheme operates imagine the simplest possible distortion of space: a section of the $x-$ axis is compressed as shown in the figure below. We probe the compressed region with two rays in order to find the values of $\boldsymbol{\varepsilon}(\mathbf{r})$ and $\boldsymbol{\mu}(\mathbf{r})$ that would give rise to the ray trajectory shown. We recognize that:

- $\boldsymbol{\varepsilon}(\mathbf{r}), \boldsymbol{\mu}(\mathbf{r})$ are tensors because we have singled out the *x*-axis for compression,
- in the uncompressed regions there is no change so $\boldsymbol{\varepsilon}(\mathbf{r}) = \boldsymbol{\mu}(\mathbf{r}) = 1$ in these regions,
- $\boldsymbol{\varepsilon}(\mathbf{r})$ and $\boldsymbol{\mu}(\mathbf{r})$ appear on the same footing because of the symmetry between electric and magnetic fields.

It follows from the last assertion that $\boldsymbol{\varepsilon}(\mathbf{r}) = \boldsymbol{\mu}(\mathbf{r})$.



Top: a simple coordinate transformation than compresses a section of the $x-$ axis. As a result rays follow a distorted trajectory shown on the top right but emerge from the compressed region traveling in exactly the same direction with the same phase as before. Bottom: requiring that a ray pass through the compressed region with the same phase change as through uncompressed space enables us to predict the metamaterial properties that would realize this trajectory for a ray.

Next consider a ray propagating parallel to the $x-$axis: in order to arrive at the far side of the compressed region with the same phase as in the uncompressed system we require $k'md = k_0 d$ where k_0 is the free space wave vector, k' is the wave vector in the compressed region, m is the compression factor, and d is the original thickness of the layer. Since $k' = k_0\sqrt{\varepsilon_y \mu_y}$, where ε_y and μ_y are the components of the respective tensors perpendicular to the $x-$axis, then we deduce that,

$$\varepsilon_y = \mu_y = m^{-1} \tag{1}$$

On the other hand rays propagating perpendicular to the $x-$axis travel through uncompressed space, and therefore their wave vector, k'', must take the free space value if the correct phase evolution is to be followed. In this case,

$$k'' = k_0\sqrt{\varepsilon_y \mu_x} = k_0\sqrt{\varepsilon_x \mu_y} = k_0 \tag{2}$$

and therefore using (8) we have,

$$\varepsilon_x = \mu_x = m \tag{3}$$

Also: because $\boldsymbol{\varepsilon}(\mathbf{r}) = \boldsymbol{\mu}(\mathbf{r})$, the compressed layer is impedance matched and does not reflect.

The above gives an intuitive version of our scheme. A more formal derivation was presented by Ward and Pendry and an updated version by Schurig et al using modern notation. We follow the latter version here. If the distorted system is described by a coordinate transform $x'^{j'}\left(x^j\right)$ we define,

$$\Lambda_j^{j'} = \frac{\partial x^{j'}}{\partial x^j} \tag{4}$$

Then in the new coordinate system we must use modified values of the permittivity and permeability to ensure that Maxwell’s equations are satisfied,

$$\begin{aligned} \varepsilon'^{i'j'} &= \left[\det(\Lambda)\right]^{-1} \Lambda_i^{i'} \Lambda_j^{j'} \varepsilon^{ij} \\ \mu'^{i'j'} &= \left[\det(\Lambda)\right]^{-1} \Lambda_i^{i'} \Lambda_j^{j'} \mu^{ij} \end{aligned} \tag{5}$$

We go on to describe the rich variety of devices that can be produced by application of TO. I reproduce just one figure below which shows variously: a design for a beam splitter, and a cylindrical to planar wave transformer.

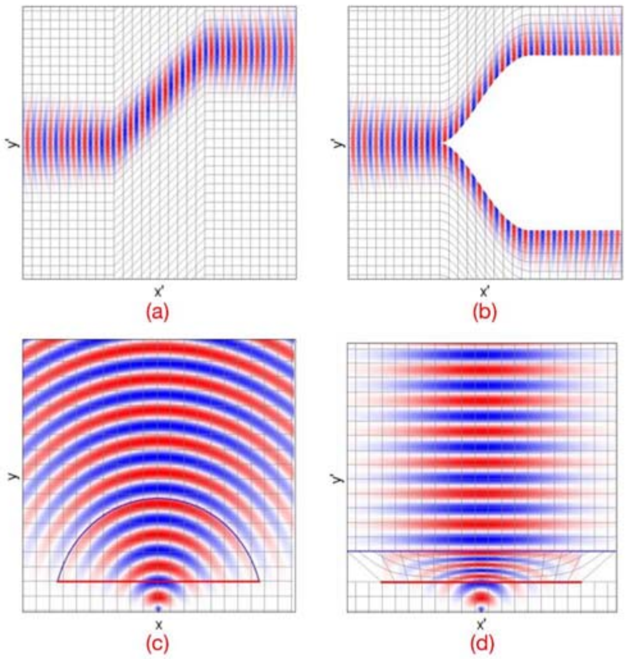


Fig. 8. *Three finite-embedded transformation are shown. In (a), a beam is shifted laterally as it passes through bream-shifting media. In (b), two beam shifters have been placed side by side to design a beam splitter. In the image shown soft boundary conditions have been placed on the internal boundaries. In (c) and (d), an embedded transformation which converts cylindrical waves to planar waves is shown.*

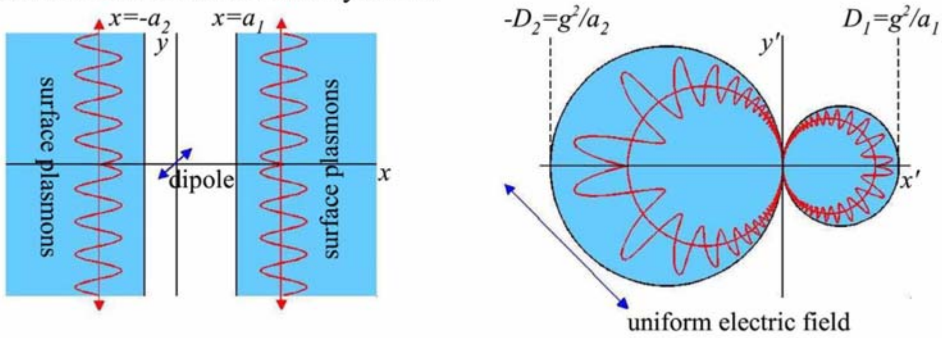
In the pipeline are further collaborations. For example we have hired as faculty here are Imperial College on of Xiang Zhang’s post docs, Rupert Oulton (also a graduate of Imperial) a move that will be a strong stimulus to collaborative work on plasmonics. My team at Imperial College, using TO has developed detailed understanding at the analytic level of a whole family of plasmonics devices designed to harvest light and concentrate it into a very small sub wavelength volume as documented in papers [5,7,8,10].

We illustrate the concept below where a well understood system of a dipole source radiating energy into a waveguide formed by two slabs of silver is transformed into a totally different geometry: that of two kissing cylinders. Whereas the slabs harvest light from a local dipole radiator and distribute the energy to infinity, in contrast the

cylinders harvest light from an incident plane wave and transport the energy to the point of contact, greatly concentrating the energy density in the process.

Conformal transformations

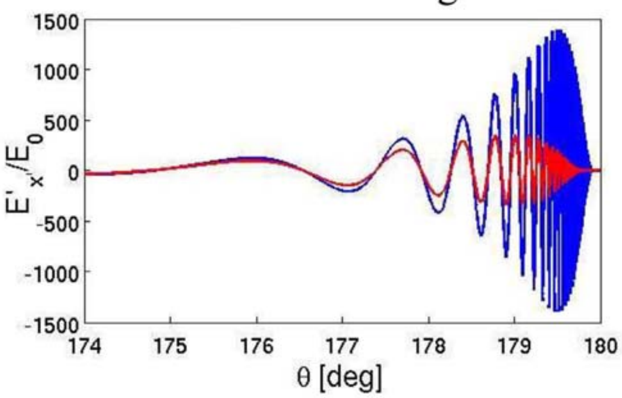
A broadband light harvesting device: inversion about the origin, $z' = 1/z$, converts two slabs into two cylinders



Left: two *semi-infinite* metallic slabs separated by a thin dielectric film support surface plasmons that couple to a dipole source, transporting its energy to infinity. The spectrum is continuous and broadband therefore the process is effective over a wide range of frequencies.
Right: the transformed material now comprises two *finite* kissing cylinders. The dipole source is transformed into a uniform electric field.

This concentration of energy, a consequence of the plasmon waves moving slower and slower as they move towards the touching point, is illustrated in the figure below which shows field enhancements of the order of 10^3 implying energy densities enhanced by a factor approaching 10^6. Experiments on these systems are currently in progress in Berkeley, at Duke and will shortly be implemented at Imperial College.

Field enhancement versus angle



Blue curve: E_x at the surface plotted as a function of θ, for $\omega = 0.75\omega_{sp}$ and $\varepsilon = -7.058 + 0.213i$ taken from Johnson and Christy.
Red curve: $\varepsilon = -7.058 + 2 \times 0.213i$ i.e. more loss. Both curves are normalised to the incoming field amplitude *E*0.

It is evident from the above that more there has been considerable exchange of ideas between the US groups and my group in London. This continues, and further exchange of personnel is planned for 2011.

JB Pendry
Imperial College London
10 February 2011

List of Publications associated with my group in 2010

1. Three-Dimensional Invisibility Cloak at Optical Wavelengths
   Tolga Ergin, Nicolas Stenger, Patrice Brenner, John B. Pendry, and Martin Wegener
   Science **328**, 337 (2010).

2. Quantum friction-fact or fiction?
   JB Pendry
   New Journal of Physics, **12** 033028 (2010).

3. Super phase array
   WH Wee, JB Pendry
   New Journal of Physics, **12** 033047 (2010).

4. Looking beyond the perfect lens
   WH Wee, JB Pendry
   New Journal of Physics **12** 053018 (2010).

5. Plasmonic Light-Harvesting Devices over the Whole Visible Spectrum
   A. Aubry, DY Lei, AI Fernandez-Dominguez, S. Maier and JB Pendry
   Nano Letters, **10,** 2574-2579, (2010).

6. Cross-section comparisons of cloaks designed by transformation optical and optical conformal mapping approaches
   Yaroslav A Urzhumov, Nathan B Kundtz, David R Smith & John B Pendry
   Journal of Optics, **13,** 024002, (2011).

7. Broadband plasmonic device concentrating the energy at the nanoscale: The crescent-shaped cylinder
   Alexandre Aubry, Dang Yuan Lei, Stefan A. Maier, and J. B. Pendry
   Physical Review, **B82**, 125430, (2010).

8. Conformal transformation applied to plasmonics beyond the quasistatic limit
   Alexandre Aubry, Dang Yuan Lei, Stefan A. Maier, and J. B. Pendry
   Physical Review, **B82**, 205109, (2010).

9. Electromagnetic design with transformation optics
   Nathan B. Kundtz, David R. Smith, and John B. Pendry
   Proceedings of the IEEE,.

10. Surface Plasmons and Singularities
    Y. Luo, JB Pendry, A. Aubry A
    Nano Letters, **10** 4186-4191 (2010).